MANMOHAN TECHNICAL UNIVERSITY
OFFICE OF THE CONTROLLER OF EXAMINATIONS
MODEL QUESTION, 2080 Asar

Level: Bachelor | **Year/Part: I/II**
Faculty: School of Engineering | **F.M.: 50**
Program: Civil/Electrical &Electronics | **P.M.: 20**
Subject: Engineering Math II (EG451SH) | **Time: 3 Hours**

- Group A contains Multiple Choice Questions.
- Candidates are required to give their answers in their own words.
- The figure in the margin indicate Full Marks.
- Attempt all Questions.

Group A $[10 \times 1 = 10]$

1. The value of $x\frac{\partial U}{\partial x} + y\frac{\partial U}{\partial y}$ if $U = \frac{(\sqrt{x}+\sqrt{y})sin^{-1}(\frac{y}{x})}{x^3+y^3}$ is
   a) -2.5U b) -1.5U c) 0 d) -0.5U
2. The area of parallelogram determined by vectors $\vec{\imath} + 2\vec{\jmath} + 3\vec{k}$ and $3\vec{\imath} - 2\vec{\jmath} + \vec{k}$ is
   a) 8 b) $8\sqrt{3}$ c) $4\sqrt{3}$ d) none
3. If $\vec{a}$ is constant vector and $\vec{r}$ is position vector then the value of $\nabla.(\vec{a} \times \vec{r})$ is
   a) $2\vec{a}$ b) $-2\vec{a}$ c) 0 d) 3
4. The series $\sum_{n=1}^{\infty}(-1)^{n-1}\frac{1}{\sqrt{2n+1}}$ is
   a) conditionally convergent c) absolute convergent
   b) Divergent d) oscillating
5. The value of $\int_{-1}^{1}\int_{0}^{z}\int_{x-z}^{x+z}(x+y+z)dydxdz$, is
   a) 4 b) -2 c) 0 d) 5
6. The equation of plane through (-11, 4, -2) with normal vector $6\vec{\imath} - 5\vec{\jmath} - \vec{k}$ is
   a) 6x-5y-z+84 = 0 c) 6x+5y+2z+1 = 0
   b) -6x+5y+z+20 = 0 d) 6x+4y-z+94 = 0
7. If the plane 2x-y+z = 0 is parallel to line $\frac{2x-1}{2} = \frac{2-y}{2} = \frac{z+1}{a}$; the value of a is
   a) 4 b) 2 c) -2 d) -4
8. The section of sphere by the plane is
   a) Circle b) parabola c) Hyperbola d) ellipse
9. For the Bessel's function $J_n(x)$, the value of $[J_{\frac{1}{2}}(x)]^2 + [J_{\frac{-1}{2}}(x)]^2 =$
   a) $\sqrt{\frac{2}{\pi x}}$ $b)\ \frac{2}{\sqrt{\pi x}}$ $c)\sqrt{\frac{2}{\pi}}x$ $d)\ 1$
10. For the Legendre polynomial $P_n(x)$ which is not true
    a) $P_n(1) = 1$ b) $P_0(x) = 0$ c) $P_0(x) = 1$ d) $P_n(-x) = (-1)^n P_n(x)$

Group B

Attempt any EIGHT questions $[\ 8 \times 2 = 16]$

11. Find Unit normal vector to the surface $z - x^2 - y^2 = 0$ at the point (-1, -2 , 5).
12. For the Bessel's function $J_n(x)$, prove that $J_{\frac{-1}{2}}(x) = \sqrt{\frac{2}{\pi x}}\ cosx$.
13. Find the image of point (1,3,4) in the plane $2x - y + z + 3 = 0$
14. Show that the lines $\frac{x+1}{-3} = \frac{y-3}{2} = \frac{z+2}{1}\ and\ \frac{x}{1} = \frac{y-7}{-3} = \frac{z+7}{2}$ are coplanar.
15. Obtain the equation of sphere through circle $x^2 + y^2 + z^2 = 9$, $x - 2y + 2z = 5$ as a great circle.
16. Obtain the equation of right circular cylinder of radius 4 and axis is the line $x = 2y = -z$.
17. Find by double integration the area lying between curve $y = 4x - x^2$ and line $y = x$.
18. If $U = \ sin^{-1}\frac{x^2+y^2}{x+y}$ Prove that $\frac{\partial U}{\partial x} + y\frac{\partial U}{\partial y} = tanU$.
19. xpress $f(x) = x^3 - 5x^2 + x + 2$ in terms of Legendre's Polynomials.

Group C $[6 \times 4 = 24]$

20. Find the radius and interval of convergence of the power series $\sum_{n=0}^{\infty}(-1)^n\frac{(x-3)^n}{n+1}$.
21. Find the maximum value of xyz under the condition $x + y + z = 8$.
22. Evaluate $\int_0^a\int_0^{\sqrt{a^2-x^2}} y^2\sqrt{x^2+y^2}dydx$ by changing to polar coordinates.
23. Solve the differential equation $y'' - 4xy' + (4x^2 - 2)y = 0$ by power series method.
24. Prove that the necessary and sufficient condition for the vector function $\vec{a}$ of scalar variable t has constant magnitude is $\vec{a}.\frac{d\vec{a}}{dt} = 0$.

    OR

    If $\vec{r} = x\vec{\imath} + y\vec{\jmath} + z\vec{k}$ and $\vec{a}, \vec{b}$ are constant vectors then prove that

    $$\text{Curl}\ [\vec{r} \times (\vec{a} \times \vec{b})] = 2(\vec{b} \times \vec{a})$$

25. Find the magnitude and equation of the line of S.D between lines $\frac{x-3}{1} = \frac{y-5}{2} = \frac{z-7}{-3}$ and $\frac{x+1}{3} = \frac{y+2}{-4} = \frac{z+3}{1}$.

    OR

    Find the equation of tangent planes to the sphere $x^2 + y^2 + z^2 + 6x - 2z + 1 = 0$ which passes through the line $x + z - 16 = 0$, $2y - 3z + 30 = 0$.

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